Workshop on

Equilibrium Problems and Related Topics

May 20–21, 2022

Department of Information Engineering, Computer Science and Mathematics
University of L'Aquila

Organizing Committee

Monica Bianchi, Dipartimento di Matematica per le Scienze economiche, finanziarie ed attuariali, Università Cattolica del Sacro Cuore di Milano

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UNIVERSITÀ CATTOLICA del Sacro Cuore

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Program

Friday, May 20

Chairman: *Massimiliano Giuli*

14:00-14:45 *Juan Enrique Martínez-Legaz* (Universitat Autònoma de Barcelona, España)

▷ Duality for quasiconvex minimization over closed convex cones

14:45-15:30 *Monica Milasi* (Università degli Studi di Messina, Italia)

▷ Quasi-variational problems with non-self map on Banach spaces: existence and application

15:30-16:00 Coffee break

Chairman: *Monica Bianchi*

16:00-16:45 *John Cotrina* (Universidad del Pacífico, Lima, Perú) - on a conference call

▷ Remarks on the Berge maximum theorem

16:45-17:30 *Fabian Flores-Bazan* (Universidad de Concepción, Chile) - on a conference call

▷ A notion of conjugacy for one-dimensional set-valued mappings

Saturday, May 21

Chairman: *Rita Pini*

09:30-10:15 *Nicolae Popovici* (Universitatea Babeş-Bolyai, Cluj-Napoca, Romania) - on a conference call

▷ Characterizations of solutions to equilibrium problems via extreme or exposed points

10:15-11:00 *Nicholas Hadjisavvas* (University of the Aegean, Ermoupolis, Syros, Greece)

▷ Quasiconvex families of functions and quasimonotone families of operators

11:00-11:30 Coffee break

Chairman: *Marco Castellani*

11:30-12:15 *Mihaela Miholca* (Universitatea Tehnică, Cluj-Napoca, Romania)

▷ Well-posedness for set-valued equilibrium problems

12:15-13:00 *Simone Sagratella* (Università di Roma "Sapienza", Italia)

▷ Solution methods for generalized Nash equilibrium problems with mixed-integer variables

Abstracts

Remarks on the Berge maximum theorem

John Cotrina

The Berge maximum theorem is an important tool in the area of general equilibrium theory. In this work, we complement a maximum theorem proposed by Morgan and Scalzo under pseudo-continuity. We also show that a generalized Nash equilibrium problem can be reformulated as a classical Nash game under suitable assumptions. Moreover, the existence result of Nash equilibria proposed by Morgan and Scalzo can be deduced by Debreu's theorem in the finite dimensional setting.

A notion of conjugacy for one-dimensional set-valued mappings

Fabian Flores-Bazan

It is known that the classical convex duality theory as well as some nonconvex duality schemes are based on the notion of convex conjugate function. We propose a notion of conjugate mapping for multivalued functions $F : X \rightrightarrows \mathbb{R}$, and derive some of its main properties, which we expect to be useful in optimization theory.

Quasiconvex families of functions and quasimonotone families of operators

Nicolas Hadjisavvas

One of the features of quasiconvex functions is that, in contrast to convex functions, quasiconvexity is not preserved under addition. Similarly, the sum of two quasimonotone operators is not quasimonotone in general. In this work, we show that if the functions belong to what we call a quasiconvex family, then their sum (and also the pointwise minimum) is quasiconvex. An analogous situation occurs with quasimonotone operators. We define quasimonotone families of operators, and we examine how they behave under addition. In particular we also examine what happens with strict or semistrict quasimonotonicity.

Some particular attention is given to properly quasimonotone operators, because of their importance. We especially describe the case when a quasimonotone operators is not properly quasimonotone.

Duality for quasiconvex minimization over closed convex cones

Juan Enrique Martinez-Legaz and Wilfredo Sosa

In this joint paper with Wilfredo Sosa, we establish a general duality theorem in a generalized conjugacy framework, which generalizes a classical result on the minimization of a convex function over a closed convex cone. Our theorem yields two quasiconvex duality schemes; one of them is of the surrogate duality type and is applicable to problems having an evenly quasiconvex objective function, whereas the other one is applicable to problems with Lipschitz quasiconvex objective functions and yields duals whose objective functions do not involve any surrogate constraint.

Well-posedness for set-valued equilibrium problems

Mihaela Miholca

In this paper we extend a concept of well-posedness for vector equilibrium problems to the more general framework of set-valued equilibrium problems in topological vector spaces using an appropriate reformulation of the concept of minimality for sets. Sufficient conditions for well-posedness are given in the generalized convex settings and we are able to single out classes of well-posed set-valued equilibrium problems. On the other hand, in order to relax some conditions, we introduce a concept of minimizing sequences for a set-valued problem, in the set criterion sense, and further we will have a concept of well-posedness for the set-valued equilibrium problem we are interested in. Sufficient results are also given for this well-posedness concept.

Quasi-variational problems with non-self map on Banach spaces: existence and application

Monica Milasi

We focus on the analysis of generalized quasi-variational inequality problems with a non-self constraint map. To study such problems, we consider the concept of the projected solution and we prove the existence in real Banach spaces. Following the stochastic variational approach introduced by Rockafellar and Wets, we introduce the concept of the projected solution in the stochastic setting, and we prove the existence of such a solution. We apply these theoretical results in studying an electricity market with renewable power sources.

Characterizations of solutions to equilibrium problems via extreme or exposed points

Nicolae Popovici and Valerian-Alin Fodor

We consider scalar equilibrium problems governed by a bifunction in a finite-dimensional framework. By using classical arguments in Convex Analysis, we show that under suitable generalized convexity assumptions imposed on the bifunction, the solutions of the equilibrium problem can be characterized by means of extreme or exposed points of the feasible domain. Our results are relevant for different particular instances, such as variational inequalities and optimization problems, especially for best approximation problems.

Solution methods for generalized Nash equilibrium problems with mixed-integer variables

Simone Sagratella

A multi-agent system in many practical noncooperative frameworks can be modeled as a generalized Nash equilibrium problem (GNEP). Many solution methods have been proposed in the literature to compute solutions, or equilibria, of a GNEP. However, most of them are based on reformulating the GNEP as a suitable variational inequality. Such reformulations are not relevant if any agent's optimization problem comprises some discrete variables. Indeed, in the resulting mixed-integer setting, in order to solve the GNEP, it is necessary to consider alternative methods that directly refer to the definition itself of equilibrium.

With this in mind, we consider wide classes of GNEPs with mixed-integer variables that can be provably solved by using suitable best-response methods. Moreover, we define some branch-and-bound type algorithms to solve these GNEPs with mixed-integer variables.

Most of the results presented can be found in a bunch of recently published papers, while others come from ongoing research.